Mathematics for Economists
module 1, academic year 2021–2022

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Course description

The course "Mathematics for Economists" is designed to introduce the students to mathematical and programming tools which are widely used in economics, particularly in micro- and macroeconomics lecture courses. The course is an elective one, and is taught at the first module of the first year. It consists of 14 lectures and 7 seminars.

Course requirements, grading, and attendance policies

The course doesn't have any special prerequisites except for the standard calculus and linear algebra courses.

There will be 5 home assignments which will constitute 20% of the final grade. The final exam will account for the remaining 80%.

Course contents

1. Preliminaries
   (a) The intermediate and mean value theorems
   (b) The inverse and implicit function theorems
2. Finite-dimensional optimization
   (a) Unconstrained optimization problem
   (b) Equality-constrained optimization problem, theorem of Lagrange
   (c) Convexity and optimization
3. Parametric optimization and comparative statics
   (a) Monotone comparative statics
   (b) Continuous comparative statics
4. Multicriteria optimization. Pareto optimum. Application to simple games.
5. Linear algebra

   (a) Linear space $\mathbb{R}$
   (b) Systems of linear equations, linear subspaces
   (c) Symmetric matrices, eigenvalues, quadratic forms

6. Probability
   (a) Probability space. Unconditional and conditional probability. Independence
   (b) Law of total probability. Bayes's formula. Random variables. Distribution
   (c) Characteristics of a random variable. Discrete and continuous random variables
   (d) Random vectors. Joint distribution. Conditional distribution
   (e) Law of large numbers. Central limit theorem

7. Statistics
   (a) Estimation of parameters. Properties of estimators
   (b) Confidence intervals
   (c) Hypotheses testing. Type one and type two errors

Description of course methodology

Lectures will proceed from motivating examples and sample models in economics to general principles of mathematical modeling.

Sample tasks for course evaluation

1. Find the set of all solutions in the following optimization problem as a function of $w > 0$:

$$\max x^{1/4}y^{1/4} + z \quad \text{s.t. } w - x - y - z \geq 0,\ x, y, z \geq 0.$$

2. Solve the following optimization problem as a function of $\alpha > 0$:

$$\max \alpha x + y \quad \text{s.t. } y + (x-1)^3 \leq 0,\ x, y \geq 0.$$

3. Quadratic form in $\mathbb{R}^3$ is defined by the following symmetrical matrix:

$$\begin{pmatrix} \alpha & 1 & 1-\alpha \\ 1 & 2 & 0 \\ 1-\alpha & 0 & 1 \end{pmatrix}.$$

   Find the values of α for which the form is positive semidefinite.

4. On each bet, a gambler loses 1 with probability 0.7, loses 2 with probability 0.2, or wins 10 with probability 0.1. Approximate the probability that the gambler will be losing after his first 100 bets.

5. A company that manufactures brackets for an automaker regularly selects brackets from the production line and performs a torque test. The goal is for mean torque to equal 125. Let X equal the torque and assume that X is $N(\mu, \sigma^2)$. We shall use a sample of size $n = 15$ to test $H_0 : \mu = 125$ against a two-sided alternative hypothesis.

(a) Give the test statistic and a critical region with significance level $\alpha = 0.05$. Sketch a figure illustrating the critical region.

(b) Use the following observations to calculate the value of the test statistic and state your conclusion:

128 149 136 114 126 142 124 136 122 118 122 129 118 122 129

Course materials

1. Rudin, W. (1976) Principles of Mathematical Analysis, Third Edition, McGraw-Hill International Editions, Singapore.
2. Sundaram R. K. (1996), A First Course in Optimization Theory, Cambridge University Press.
3. Ehrgott M. (2005) Multicriteria Optimization, Second edition, Springer.
4. Osborne M. J. (2003) An Introduction to Game Theory, Oxford University Press.
5. Gantmacher F. (1959) Theoryof matrices, AMS Chelsea publishing.
6. Ross S. (2014) A first course in probability, Ninth edition, Pearson.
7. Hogg R. V., Tanis E. A., Zimmerman D. L., Probability and statistical inference, Ninth edition, Pearson.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.